# COULD GOVERNMENT ACHIEVE SOCIAL OPTIMUM FROM PUBLIC-PRIVATE PARTNERSHIP IN URBAN TRANSPORTATION INFRASTRUCTURE MANAGEMENT FOR FREE? A MECHANISM DESIGN APPROACH

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**ABSTRACT**

Highway system is vital in urban transportation network. Operating, maintaining and financing of existing highway (a.k.a. brownfield highway) are increasingly important. Public-Private Partnership(PPP) is implemented in various countries to solve problems in urban transportation infrastructure projects. In this paper, we will mathematically model PPP to discuss whether government could achieve social optimum for free in urban highway PPP project. Previous modelling methods in PPP share two common shortcomings: over-simplified assumption on behavior of users and owners of the highway and insufficient attention to privately-held information. In this paper, we create a new form of PPP, improved Investment Public-Private Partnership using mechanism design as a multi-leader-multi-follower (MLMF) Stackelberg game. The implementibility in MLMF Stackelberg game in dominant strategy and Bayesian equilibria are derived and the feasibility of the model is proved through these theorems. The condition is discussed and given on achieving social optimal and budget balance simultaneously, that is the government gets job done for free. The Sioux-Falls network model is used to illustrate and verify the model.

## BIOGRAPHICAL SKETCH

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Bingyan's thesis was supervised by Prof. Oliver H. Gao.

This thesis is dedicated to Yifan and Zhiping for giving me life, raising me up,
and loving me unconditionally,
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CHAPTER 1

# INTRODUCTION

Nowadays, urban transportation system is fundamental in American transportation system. The highway system play the key role in urban area. It connects major population centers, public transportation facilities, and different transportation destinations. However, such system is suffering from several problems in both supply and demand sides. On the demand side, traffic congestion harms logistic efficiency and causes enormous economic losses. In 2011, congestion in America costs drivers $121 billion and 5.5 billion additional hours on the road [13]. In 2012, 42% of America's major urban highways remained congested [21]. Meanwhile on the supply side, the government is lack of fund and expertise in operating and maintaining the highways. Although the investment increased to $91 billion annually in 2012, it is still insufficient compared with Federal Highway Administration's estimation of $170 billion annually in capital investment that is needed for significant improvement in pavement conditions [21].

Therefore, designing, building, financing, operating and maintaining issues in highway system are intensely investigated [4][27][1]. There are three kinds of highway projects, greenfield highway project, which focuses on constructing new highways, brownfield highway project, which focuses on managing existing highways, and hybrid highway project, which is a mix of former two projects. Compared with greenfield highway projects, brownfield projects are more complicated because they draw more public concerns and hence are facing more political obstacles [22]. Currently, the mainstream of highway management projects in U.S. is government provision (i.e. the government takes charge of the whole project), which is already proven to be insufficient [4].

One popular and prevailing solution to that is Public-Private Partnership (PPP). PPP has different definitions in different countries [23]. In the US, it is defined as "a contractual agreement between a public agency (federal, state or local) and a private sector entity" through which the skills, assets, rewards and risks "are shared in delivering a service or facility for the use of the general public" [7]. Such projects are heavily utilized in the past decades in various countries. As one of the primary incentives to apply PPP is saving government spending budget, one question should be ask before kick off any project:

Could government achieve successful PPP project (i.e. social optimum) for free?

To answer this question, mathematical modelling of PPP is needed. Many researchers have attempted to model various aspects of the projects. Previous methods include mathematical programming, game theory and simulation.

Mathematical programming is widely used in studying risk allocation, bundling issues, investment, etc. Iossa et al.[14] presented a method mainly based on optimization to compare the performance of PPP ("referred as bundling building and operation into a single contract") and public provision under different conditions and assumptions. They pointed out that PPP is a desired project form if the externality between building and operation is positive. Engel et al. [6] provided a thorough investigation in PPP financing, including project finance, investment cost, role of government, and renegotiation.

Game theory is used to understand the interaction among different sectors [30]. Among them, Medda [19] studied the allocation of risk between government and private sector, Ho [11] modeled the financial renegotiation between government and private sector, and Zou *et al.* [30] developed a game theory model to understand the dynamic relationships between government and private sector. Verhoef [24] evaluated the effect of auction on privatization of road and provided various 'indicators' that government may use to optimize the bid.

Simulation is also used in the analysis of PPP. In Zhang et al. [29], an agent-based simulation model was built to evaluate the competition between privatized road and public road and help making pricing and capacity decisions. Zhang [28] modeled the privatization of road network in an evolutionary model. It is the first time to consider pricing, investment and ownership as a whole.

There are also other methods which were used in modeling PPP. Martimort et al. [18] applied task assignments in discussion of whether or not to bundle the construction and management of infrastructure. Kang et al. [16] assessed risks in PPP project using utility theory. Other theories, such as fuzzy set theory [5], are also used in identification and measurement of risks in PPP project.

All previous works attempting to model PPP projects share two common shortcomings:

**Firstly, almost all previous works put simplified assumption on the users of the road network.** Some assumed that the demand is exogenous or inelastic and some assumed that users of the road network, who is an important component of the whole system, will not play strategically. However, some literature in transportation analysis suggested that it is necessary to consider users of the road as multiple players or a single player as a whole in the game theoretical model to achieve more realistic and accurate results [12].

In this paper, we use Stachelberg game to model PPP project. Stackelberg game is a non-cooperative decision making problems where there are two groups of players: the players who hold the powerful positions in the game are referred as the leaders and other players who react to the leaders' decisions are called the followers [3]. In PPP project, private sectors, the leaders, first determine the toll rate on all the roads they have rented, and then public sectors, the followers, observing the toll rate in the network, make their decisions to travel on the road. As both private sectors and public sectors are selfish utility maximizers, this game is a non-cooperative multi-leader-multi-follower (MLMF) Stackelberg game. By analyzing this game, we could predict the behavior of both groups of players and evaluate the outcome of PPP project.

**The second one is paying insufficient attention to privately held information.** In a PPP project, one should admitted that some information necessary for decision making is privately held, given maintenance effort or cost for example. It is important to obtain this private information if the government wants to implement an efficient and successful PPP project. Nonetheless, one should also realize that revelation of information is not free [15]. In the point of view of game theory, every player would play strategically to maximize their own benefit or minimize their own cost, but such strategy usually may not be beneficial, and sometimes may be harmful, to obtaining social optimal in the system. For example, private sector has incentive to raise the toll rate to make the project more profitable, and the price is usually higher than the social optimal one.

Our paper introduces mechanism design into PPP modeling to overcome such obstacle. Mechanism design is a theory that focuses on design of institutions to achieve certain objectives, with the assumption that all the players will act strategically and hold private information necessary in decision making. Although there are many advanced tools had already been used in modeling, analyzing and evaluating PPP projects, mechanism design is not paid enough attention. Only [14] used mechanism design to set up a truth-telling mechanism to prevent cost overruns. However, applying mechanism design in PPP will enable us to deal with privately held information and hence to model the whole project and determine key parameters more precisely, directly and easily.

In mechanism design, implementability is a key concept which indicates whether a desired outcome could be achieved in a given equilibrium through the mechanism [15]. There are plenty of literature investigating in the dominant strategy incentive compatibility (DSIC) and Bayesian incentive compatibility (BIC) in single level game [26], but discussion of DSIC and BIC in Stackelber game context is insufficient. Only [8] addressed the single-leader-multi-follower Stackelberg problem. We provides some sufficient and necessary conditions of DSIC and BIC in MLMF Stackelberg game.

In this paper, we are going to answer the question 'could government manage transportation infrastructure for free?' To do this, we will study PPP project through mechanism design in a MLMF Stackelberg game. However, there are two major problems when PPP is utilized in brownfield highway project: the first is unrealistic price, that the price private sector charge is usually higher than optimal level [22]. Government needs to regulate the private sector carefully in the contract. Another problem is lack of public support. Collecting toll on existing unpriced road usually incurs unsatisfactory from the users and owners of the road, especially from residences who travel on the road in daily commute. This is often a major public concern in utilizing PPP approach.

Investment Public-Private Partnership, aiming at resolving political discrepancy in public, had been proposed recently [10]. Ideas that similar with IPPP had achieved great success in management of local oil resources in Alaska. Geddes et al.[10] stated that IPPP will increase public support on road pricing, reduce income inequality among different families, and increase household income. Although [10] using existing projects that are similar to IPPP to illustrate the feasibility and advantages of IPPP, direct quantitative analysis is still absent to verify their claims.

**Main contributions** In this paper our contribution could be summarized in two parts. On methodology side, we provide the condition of a mechanism to be DSIC and BIC in a MLMF Stackelberg game. Using these results, we mathematically design and model a new form of PPP project, improved IPPP, which is based on IPPP. On application side, by analyzing and evaluating the mathematical properties of improved IPPP, we derive the condition when system optimal and budget balance could coexist in PPP project. This answers the question: could government get PPP done for free? We also compare improved IPPP with government provision, PPP and original IPPP and conclude that improved IPPP has advantages in achieving potential Pareto-improvement, relief of tax distortion, public support, better regulation on road pricing and lower financial risk. To our best knowledge, there is no other literature comprehensively modeling transportation infrastructure project management in mechanism design. Our paper would provide insights for research in both mechanism design and infrastructure project management.

In the following, section 2 explains IPPP and our model of improved IPPP, section 3.1 derives some general and useful results in implementability in mechanism design context, section 4 applies these results on our model and proves the feasibility of improved IPPP, section 5 shows an example of implementing IPPP in Sioux-Falls network and section 6 serves as conclusion and provides some possible direction in future work.

CHAPTER 2

# MODELLING IPPP WITH OPTIMAL MECHANISM DESIGN IN MULTI-LEADER-MULTI-FOLLOWER (MLMF) STACKELBERG GAME

## 2.1 IPPP and improved IPPP

In an IPPP project, there are three participants:

- Government: Government takes charge in operating the IPPP project, including holding auction, setting up lease contract with private sector, collecting rent of highway and managing permanent fund;
- Private sector: Private sector rents highway from government and then becomes responsible for financing, operating and maintaining the highway according to the standard required in contract provision;
- Public sector: Includes all residence in the jurisdiction that possesses the highway and all travelers (users) using the highway.

As suggested by [10], an IPPP project works as follows: Government leases highways to private sectors through concession. Then private sectors operate, maintain the highway, collect toll as their income and pay the rent of highway to government. Government invests the income from highway rent and set up a permanent fund which is owned by all residence living within the jurisdiction where the highway is priced. Such fund will be managed by professional investors and dividends will be paid to all the owners.

In [10], the income of permanent fund (or rent of highway) is fixed on toll rate and traffic flow. As we pointed out, the costs of private sectors and VOTs of public sectors are unknown and stochastic. A private sector with high maintenance cost needs to raise toll price to cover her cost. Meanwhile, a private sector with low maintenance cost could also raise the price to sweeten up the profit margin. The government should allow the former case while at the same time prevent the latter case. Thus in our model, we make the rent of the highway to be floating and related to toll rate and traffic flow. This would provide both better regulatory and lower financial risks. Such merit is enabled by using of mechanism design. In order to distinguish our model to the idea in [10], we call the one in our paper 'improved IPPP' or IPPP and refer the other one 'original IPPP'. The whole framework of IPPP project in this paper is shown in Figure 2.1.

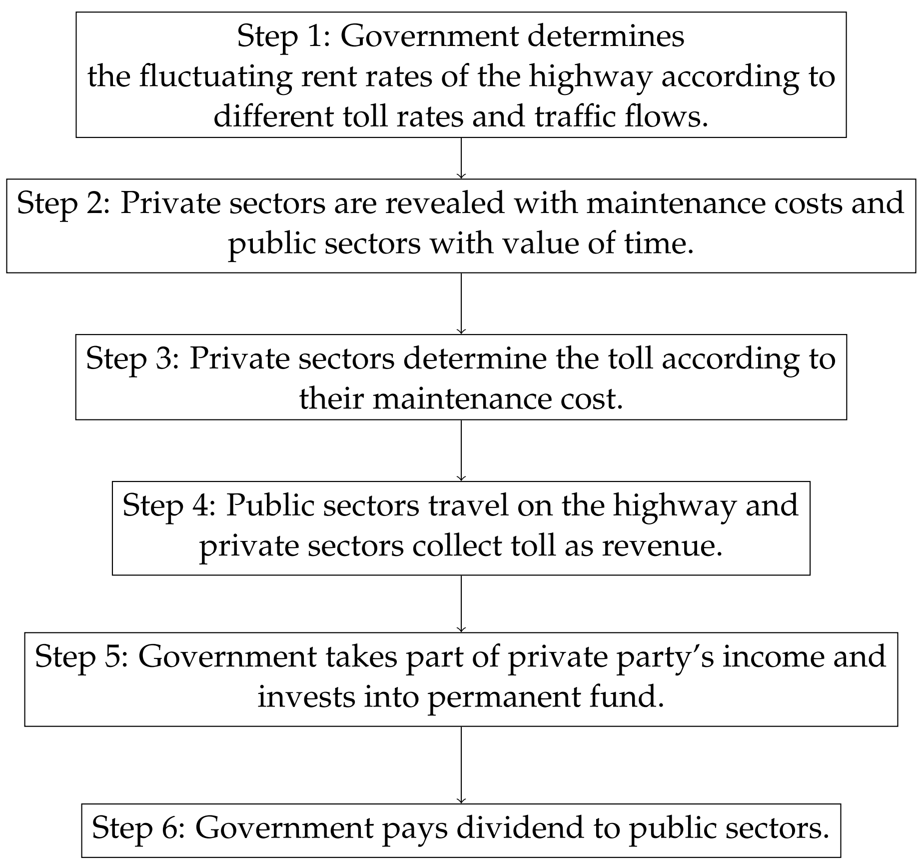


Figure 2.1: IPPP Modelling Framework

In the following sections, without further clarification, the notion 'IPPP' is refering to 'improved IPPP'.

## 2.2 An MLMF Stackelberg game

As me mentioned in section 1, the private sectors in IPPP are regarded as leaders and public sectors as followers. Let $M$ denote the number of private sectors, and $N$ of public sectors. The utility function $u_l^i$ of private sector $i = 1, \ldots, M$ is determined by maintenance cost $c^i$, the toll rate, and the flow. As there are several private sectors in the network, the utility $u_l^i$ does not only depend on $t^i$, the toll rate of leader $i$, but also the toll rate of other private sectors, $\mathbf{t}^{-i}$, and all the flow on the road, $\mathbf{f}$. Thus the utility function of private sector $i$ is $u_l^i(t^i, \mathbf{t}^{-i}, \mathbf{f} \mid c^i)$. Similarly, the utility of public sector $j$ depends on its value of time $v^j$, the toll rate $\mathbf{t}$, and flow on the network, $f^j$ and $\mathbf{f}^{-j}$. The utility function writes $u_o^j(\mathbf{t}, f^j, \mathbf{f}^{-j} \mid v^j)$.

All the players are utility maximizers. Thus private sector $i$ determines the toll rate $t^i$ by maximizing the utility function given that the maintenance cost is $c$. However, as all the decisions of private parties are made at the same time, she also needs to consider, $\mathbf{c}^{-i}$, the possible maintenance cost of other parties.

$$t_c^{i,*} = \arg\max_{t \in T} \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}} \left[ u_l^i(t, \mathbf{t}^{-i}, \mathbf{f}^*_{\mathbf{v},\mathbf{t}} \mid c) \right],$$

where $T$ is the set of all possible values $t$ (either continuous or discrete), $\mathbf{E}$ denotes expectation, $\mathbf{t}^{-i}$ is used to denote $\mathbf{t}^{-i,*}_{\mathbf{c}^{-i}}$, the best strategies of other players according to their own types for simplicity, and $\mathbf{f}^*_{\mathbf{v},\mathbf{t}}$ denotes the best response of public sectors to the toll rate $\mathbf{t}$ given the value of time is $\mathbf{v}$.

Similarly, given the tolls on all the roads are $\mathbf{t}$, the best strategy of public sector $j$ is:

$$f_{v,\mathbf{t}}^{j,*} = \arg\max_{f \in F} \mathbf{E}_{\mathbf{v}^{-j}} \left[ u_o^j(\mathbf{t}, f, \mathbf{f}^{-j} \mid v) \right],$$

where $F$ is the set of all possible value $v$ (either continuous or discrete) and $\mathbf{f}^{-j}$ is the abbreviation of $\mathbf{f}_{\mathbf{v}^{-j},\mathbf{t}}^{-j,*}$, the best response of other public sectors to the toll $\mathbf{t}$ given the value of time $\mathbf{v}^{-j}$.

Thus, to each private party $i$, the optimization problem they need to solve is:

$$\max_{t \in T} \quad \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}} \left[ u_l^i(t, \mathbf{t}^{-i}, \mathbf{f}_{\mathbf{v},\mathbf{t}}^{*} \mid c) \right] \tag{2.1a}$$

$$s.t. \quad f_{v,\mathbf{t}}^{j,*} = \arg\max_{f \in F} \mathbf{E}_{\mathbf{v}^{-j}} \left[ u_o^j(\mathbf{t}, f, \mathbf{f}^{-j} \mid v) \right] \qquad \forall : j = 1, \ldots, N \tag{2.1b}$$

This is a MLMF Stackelberg game formulated in bi-level programming.

## 2.3 Design an optimal mechanism in MLMF Stackelberg game

In problem(2.1a, 2.1b), we did not consider the role of government. Therefore the solution of(2.1a, 2.1b) is an individual utility maximal solution, rather than a social optimal solution. The $\mathbf{t}$ is the most profitable toll rate for private party and $\mathbf{f}$ is a flow that not minimizes the total travel time in the network. To make them both social optimal, government's regulatory is needed.

Let us assume that the government wants to maximize the expected social welfare, that is,

$$\max_{\mathbf{p}} \ \mathbf{E}_{c,v} SW(\mathbf{p}(\cdot))$$

Let $\boldsymbol{\tau}_\mathbf{c} = (\tau_c^1, \ldots, \tau_c^M) \in \mathbf{T}^*$ denote the social optimal toll rate given that the maintenance costs of all private sectors are $\mathbf{c}$, and $\boldsymbol{\phi}_{\mathbf{v},\mathbf{t}} \in \ \boldsymbol{\Phi}^*$ the optimal flow given that the toll rates $\mathbf{t}$ and the value of time $\mathbf{v}$. Government could change the utility functions of public sectors and private sectors by pay dividend and charge rent, respectively; that is, government should find a group of $p_l(\mathbf{t}, \mathbf{f}) > 0$ for private sector and $p_o(\mathbf{t}, \mathbf{f}) < 0$ for public sector, such that

$$\tau_c^i = \arg\max_{t \in T} \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}} \left[ u_l^i(t, \boldsymbol{\tau}^{-i}, \mathbf{f}^*_{\mathbf{v},\mathbf{t}} \mid c) - p_l^i(t, \boldsymbol{\tau}^{-i}, \mathbf{f}) \right] \quad \forall\ i = 1, \ldots, M,\ c \in C \tag{2.2a}$$

$$\phi_{v,\mathbf{c}}^j = \arg\max_{f \in F} \mathbf{E}_{\mathbf{v}^{-j}} \left[ u_o^j(\mathbf{t}, f, \boldsymbol{\phi}^{-j} \mid v) + p_o^j(\mathbf{t}, f, \boldsymbol{\phi}^{-j}) \right] \quad \forall\ j = 1, \ldots, N,\ v \in V \tag{2.2b}$$

Equations(2.2a, 2.2b) indicate a Bayesian-Nash equilibrium that all the sectors will take social optimal choice given that others do the same. Notice that $\mathbf{E}_{c,v} \left[ \sum_i u_l^i(\cdot|\mathbf{c}) + \sum_j u_o^j(\cdot|\mathbf{v}) \right]$ is the expected total utility of all agents and could serve as an indicator of total social welfare. Then the optimization problem is:

$$\max_{\mathbf{p}(\mathbf{t},\mathbf{f})} \quad \mathbf{E}_{c,v}\left[\sum_i u_l^i(\mathbf{t_c}, \mathbf{f}^*_{\mathbf{v},\mathbf{t}}|\mathbf{c}) + \sum_j u_o^j(\mathbf{t}, \mathbf{f}_{\mathbf{v},\mathbf{t}}|\mathbf{v})\right] \tag{2.3a}$$

$$s.t. \quad \tau_c^i = \arg\max_{t\in T} \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}}\left[u_l^i(t, \boldsymbol{\tau}^{-i}, \mathbf{f}^*_{\mathbf{v},\mathbf{t}} \mid c) - p_l^i(t, \boldsymbol{\tau}^{-i}, \mathbf{f})\right] \quad \forall\, i = 1, \ldots, M,\ c \in C \tag{2.3b}$$

$$\phi_{v,\mathbf{t}}^j = \arg\max_{f\in F} \mathbf{E}_{\mathbf{v}^{-j}}\left[u_o^j(\mathbf{t}, f, \boldsymbol{\phi}^{-j} \mid v) - p_o^j(\mathbf{t}, f, \boldsymbol{\phi}^{-j})\right] \quad \forall\, j = 1, \ldots, N,\ v \in V \tag{2.3c}$$

ProblemEquations (2.3a–2.3c) a tri-level linear optimization problem and could be solved through some existing solution method [25]. According to [26], it is equivalent to a single-level linear programming:

$$\max_{\mathbf{p}(\mathbf{t},\mathbf{f})} \quad \mathbf{E}_{c,v}\left[\sum_i u_l^i(\mathbf{t_c}, \mathbf{f}^*_{\mathbf{v},\mathbf{t}}|\mathbf{c}) + \sum_j u_o^j(\mathbf{t}, \mathbf{f}_{\mathbf{v},\mathbf{t}}|\mathbf{v})\right] \tag{2.4a}$$

$$s.t. \quad \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}}\left[u_l^i(\tau_c^i, \boldsymbol{\tau}^{-i}, \mathbf{f}^*_{\mathbf{v},\mathbf{t}} \mid c) - p_l^i(\tau_c^i, \boldsymbol{\tau}^{-i}, \mathbf{f})\right] \geqslant \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}}\left[u_l^i(t, \boldsymbol{\tau}^{-i}, \mathbf{f}^*_{\mathbf{v},\mathbf{t}} \mid c) - p_l^i(t, \boldsymbol{\tau}^{-i}, \mathbf{f})\right]$$
$$\forall\, i = 1, \ldots, M,\ c \in C, t \in T, \tag{2.4b}$$

$$\mathbf{E}_{\mathbf{v}^{-j}}\left[u_o^j(\boldsymbol{\tau}, \phi_{v,\mathbf{t}}^j, \boldsymbol{\phi}^{-j} \mid v) - p_o^j(\boldsymbol{\tau}, \phi_{v,\mathbf{t}}^j, \boldsymbol{\phi}^{-j})\right] \geqslant \mathbf{E}_{\mathbf{v}^{-j}}\left[u_o^j(\boldsymbol{\tau}, f, \boldsymbol{\phi}^{-j} \mid v) - p_o^j(\boldsymbol{\tau}, f, \boldsymbol{\phi}^{-j})\right]$$
$$\forall\, j = 1, \ldots, N,\ v \in V,\ f \in F \tag{2.4c}$$

The implementability of the mechanism in Bayesian-Nash equilibrium, or in other word, the feasibility of problemeqs. (2.3a–2.3c), is still unclear. In the next two sections, we first develop several theories in implementability in dominant strategy and Bayesian equilibrium in an MLMF Stackelberg game, and then use these results to prove the implementability of the mechanism.

# CHAPTER 3

# IMPLEMENTABILITY IN DOMINANT STRATEGY AND BAYESIAN EQUILIBRIUM IN AN MLMF STACKELBERG GAME

In this section, we will discuss the dominant strategy incentive compatibility (DSIC) and Bayesian incentive compatibility (BIC) in Stackelberg game with multi-leader and multi-follower. We consider a Stackelberg game with two levels in the hierarchy and multiple players in each level. We call the players in first level as leader and the second level as follower. Thus this is a multi-leader-multi-follower Stackelberg game.

For the consistency of the paper and the convenience of readers, we use same symbols here as previous sections to define a multi-leader-multi-follower Stackelberg game. Each of the symbols in this section has its interpretation in IPPP project, which is illustrated in section 2.

Let $N$ denote the number of leaders, $M$ the number of followers, and $\Gamma$ the set of all outcomes. Let $C$ denote the type space of leaders and $V$ the type space of followers. Then each player, leader or follower, would have a type of herself. For leader $i$, it is $c^i \in C$ and for follower $j$, $v^j \in V$. Let $\mathbf{c}$ and $\mathbf{v}$ denote the types of all leaders and followers, respectively. Each player has a utility function, which is $u_l^i(\alpha \mid c^i)$ for the leader $i$ and $u_o^j(\alpha \mid v^j)$ for the follower $j$, where $\alpha \in \Gamma$ is outcome of the game. Let $\mathbb{C} = C^1 \times \cdots \times C^N$ and $\mathbb{V} = V^1 \times \cdots \times V^M$.

At the beginning of the mechanism, all the players are revealed their own private types. Then the leaders first report their type (not necessary to be honest). Observing the leaders' announced types, $\tilde{\mathbf{c}}$, the followers report their types $\tilde{\mathbf{v}}$. After that the policy maker makes decision according to their reported types.

The decision consists of two parts: selection of an outcome through an outcome function $g : \mathbb{C} \times \mathbb{V} \mapsto \Gamma$ and choosing payment of each players through a payment functions $p_l : \mathbb{C} \times \mathbb{V} \mapsto \mathbb{R}$ and $p_o : \mathbb{C} \times \mathbb{V} \mapsto \mathbb{R}$ for leaders and followers, respectively.

## 3.1 DSIC in MLMF Stackelberg game

Now we first confine our discussion to dominant strategy equilibrium. An outcome function $g$ is **implementable in dominant strategies in a Stackelberg game** if:

- For each follower $j = 1, \ldots, M$, there exists a payment rule $p_o^j$ such that for all followers, given that the types of leaders are $\mathbf{c}$, we have:

$$v^j = \arg\max_{v \in V^j} u_o^j(g(\mathbf{c}, v, \tilde{\mathbf{v}}^{-j}) \mid v^j) - p_o^j(\mathbf{c}, v, \tilde{\mathbf{v}}^{-j}) \qquad \forall\ \tilde{\mathbf{v}}^{-j} \in \mathbb{V}/V^j. \tag{3.1}$$

- For each leader $i = 1, \ldots, N$, there exists a payment rule $p_l^i$ such that for all leaders, we have:

$$c^i = \arg\max_{c \in C^i} u_l^i(g(c, \tilde{\mathbf{c}}^{-i}, \tilde{\mathbf{v}}) \mid c^i) - p_l^i(c, \tilde{\mathbf{c}}, \tilde{\mathbf{v}}) \qquad \forall\ \tilde{\mathbf{c}}^{-i} \in \mathbb{C}/C^i,\ \tilde{\mathbf{v}} \in \mathbb{V} \tag{3.2}$$

Rewrite equation (3.1) and (3.2) in form of inequalities we have:

$$u_l^i(g(c^i, \tilde{\mathbf{c}}^{-i}, \tilde{\mathbf{v}}) \mid c^i) - p_l^i(c^i, \tilde{\mathbf{c}}, \tilde{\mathbf{v}}) \geqslant u_l^i(g(\tilde{c}^i, \tilde{\mathbf{c}}^{-i}, \tilde{\mathbf{v}}) \mid c^i) - p_l^i(\tilde{c}^i, \tilde{\mathbf{c}}, \tilde{\mathbf{v}}) \qquad \forall\, i = 1, \ldots, M,\ \tilde{c}^i \neq c^i \in C^i,\ \tilde{\mathbf{c}}^{-i} \in \mathbb{C}/C^i,\ \tilde{\mathbf{v}} \in \mathbb{V} \tag{3.3a}$$

$$u_o^j(g(\mathbf{c}, v^j, \tilde{\mathbf{v}}^{-j}) \mid v^j) - p_o^j(\mathbf{c}, v^j, \tilde{\mathbf{v}}^{-j}) \geqslant u_o^j(g(\mathbf{c}, \tilde{v}^j, \tilde{\mathbf{v}}^{-j}) \mid v^j) - p_o^j(\mathbf{c}, \tilde{v}^j, \tilde{\mathbf{v}}^{-j}) \qquad \forall\, j = 1, \ldots, N,\ \tilde{\mathbf{v}}^{-j} \in \mathbb{V}/V^j,\ \tilde{v}^j \neq v^j \in V^j \tag{3.3b}$$

For leader $i$, we define a directed graph $T_{g,l}(\tilde{\mathbf{c}}^{-i}, \tilde{\mathbf{v}})$ by

- There are $\mid C^i \mid$ nodes and $\mid C^i \mid (\mid C^i \mid -1)$ directed arcs between any pair of nodes;
- each node represents a type $c \in C^i$;
- and, for the arc from node $c_m \in C^i$ to $c_n \in C^i$ (here we omit the superscript $i$), the length is $u_l^i(g(c_n, \tilde{\mathbf{c}}^{-i}, \tilde{\mathbf{v}}) \mid c_n) - u_l^i(g(c_m, \tilde{\mathbf{c}}^{-i}, \tilde{\mathbf{v}}) \mid c_n)$.

For the followers, similarly, we define a directed graph $T_{g,f}(\tilde{\mathbf{v}}^{-j})$ by

- There are $\mid V^j \mid$ nodes and $\mid V^j \mid (\mid V^j \mid -1)$ directed arcs between any pair of nodes;
- each node represents a type $v \in V^j$;
- and, for the arc from node $v_m \in V^j$ to $v_n \in V^j$ (similarly we omit the superscript $j$), the length is $u_o^j(g(\mathbf{c}, v_n, \tilde{\mathbf{v}}^{-j}) \mid v_n) - u_l^i(g(\mathbf{c}, v_m, \tilde{\mathbf{v}}^{-j}) \mid v_n)$.

The following theorem is immediate from Theorem 3.4.4 in [26]:

**Theorem 1.** *If $M, N \geqslant 1$, then the following statements are equivalent:*

- *$g$ is implementable in dominant strategy in a multi-leader-multi-follower Stackelberg game.*
- *For every leader $i$ and every follower $j$, for every report $(\tilde{\mathbf{c}}^{-i}, \tilde{\mathbf{v}})$ and $\tilde{\mathbf{v}}^{-j}$, the graph $T_{g,l}(\tilde{\mathbf{c}}^{-i}, \tilde{\mathbf{v}})$ and $T_{g,f}(\tilde{\mathbf{v}}^{-j})$ does not have a finite cycle of negative length, respectively.*

If the outcome space $\Gamma$ is finite, we can redefine the type space $C$ and $V$ by $C, V \subseteq \mathbb{R}^{|\Gamma|}$ and for every $c^i \in C$ we define the $m^{th}$ element of it by the value that type $c^i$ places on outcome $m \in \Gamma$, that is $u_l^i(m \mid c^i)$. The definition of $v^i$ is similar. Then we can similarly get the following theorem directly from Theorem 4.2.12 in [26]

**Theorem 2.** *If $\Gamma$ is finite, $C, V \subseteq \mathbb{R}^{|\Gamma|}$ is closed and convex, $u_l^i(\cdot \mid c)$ is convex in $c$, $u_o^j(\cdot midv)$ is convex in $v$, and $g$ is implementable in dominant strategy in multi-leader-multi-follower Stackelberg game when restricted to every 2-dimensional subset of $C$ and $V$ for leaders and followers, respectively, then $g$ is IDS in MLMF Stackelberg game.*

Notice that Theorem 2 needs type spaces to be convex. When type spaces are all discrete and thus non-convex, we still could have IDS in a MLMF Stackelberg game under some certain conditions. First we introduce Monge matrix [20]

**Definition 1.** ***Monge matrix****[20]. A matrix $A = (a_{\alpha,\beta})_{\substack{\alpha=1,\ldots,I_1\in\mathbb{N},\\ \beta=1,\ldots,I_2\in\mathbb{N}}}$ is a Monge matrix if $\forall 1 \leqslant \alpha < \alpha' \leqslant I_1$ and $\forall 1 \leqslant \beta < \beta' \leqslant I_2$ it always has $a_{\alpha,\beta} + a_{\alpha',\beta'} \geqslant a_{\alpha,\beta'} + a_{\alpha',\beta}$*

Then we have the following theorem.

**Theorem 3.** *If $\Gamma$, $C$ and $V$ are finite, and for every $i = 1, \ldots, M$ and $j = 1, \ldots, N$, there exist an order over the outcomes in $\Gamma$, $\gamma_1, \gamma_2, \ldots$ and orders over type spaces in $C^i$ and $V^j$, $c_1^i, c_2^i, \ldots$ and $v_1^j, v_2^j, \ldots$, respectively, such that the matrix $A = (a_{\alpha,\beta})$ and $B = (b_{\alpha,\beta})$ in which $a_{\alpha,\beta} = v_l^i(\gamma_\beta \mid c_\alpha^i)$ and $b_{\alpha,\beta} = v_o^j(\gamma_\beta \mid v_\alpha^j)$ are Monge matrix, then $g$ is IDS in MLMF Stackelberg game if $g$ aligned with the Monge domains $\mathbb{C}$ and $\mathbb{V}$.*

The notion that a social-choice function is 'aligned with' a Monge domain could be found in Definition 3 in [20]. Notice that the result of Theorem 1, Theorem 2 and Theorem 3 could be immediately extended to a Stackelberg game with multiple levels and multiple players in each level, which is trivial and therefore omitted here.

## 3.2 BIC in MLMF Stackelberg game in linear environment

The property of DSIC is attractive to any mechanism designer. However, sometimes it is too good to be true and impossible to be achieved (as it is stated in Gibbard-Satterthwaite impossibility theorem) [9]. Thus it is necessary to generalize our result in section 3.1. Now we consider the Bayesian incentive compatibility in Stackelberg game in linear environment.

**Definition 2.** *Linear environment is defined as follows,*

- *The type space of each agent is an interval,* $C^i = [\underline{c}^i, \bar{c}^i]$ *for the leader* $i$ *with* $\underline{c}^i < \bar{c}^i$ *and* $V^j = [\underline{v}^j, \bar{v}^j]$ *for the follower* $j$ *with* $\underline{v}^j < \bar{v}^j$.
- *The types are independent. The probability density function* $\mathbf{Pr}(\cdot) = Pr_l^1(\cdot) \times \cdots \times Pr_l^M(\cdot) \times Pr_o^1(\cdot) \times \cdots \times Pr_o^N(\cdot)$.
- $Pr_l^i(c^i) > 0$, $\forall\ c^i \in [\underline{c}^i, \bar{c}^i]$, $i = 1, \cdots, M$ *and* $Pr_o^j(v^j) > 0$, $\forall\ v^j \in [\underline{v}^j, \bar{v}^j]$, $j = 1, \cdots, N$.
- *Each leader's utility function is*

$$u_l^i(g(\cdot)|c^i) = c^i v_l^i(g) + m_l^i + p_l^i,$$

*where $v_l^i(g)$ is the 'benefit' or 'cost' of leader and $m_l^i$ is initial endowment for the leader. And each follower's utility function is*

$$u_o^j(g(\cdot)|v^j) = v^j v_o^j(g) + m_o^j + p_o^j,$$

*where $v_o^j(g)$ is the 'benefit' or 'cost' of follower and $m_o^j$ is initial endowment for the follower.*

In the linear environment, we could extend Myerson's characterization theorem to MLMF Stackelberg game with some minor changes.

**Theorem 4.** *(Myerson's characterization theorem in MLMF Stackelberg game). In a MLMF Stackelberg game, let $\bar{v}_l^i$ and $\bar{v}_o^j$ denote the expectation of $v_l^i$ and $v_o^j$, respectively. Then a social choice function $g(\cdot)$ is BIC iff. $\forall\ i = 1, \ldots, M,\ j = 1, \ldots, N$*

- *$\bar{v}_l^i$ and $\bar{v}_o^j$ is non-decreasing;*
- *$u_l^i(c^i) = u_l^i(\underline{c}^i) + \int_{\underline{c}^i}^{c^i} \bar{v}_l^i(s)ds,\ \forall c^i$ and $u_o^j(v^j) = u_o^j(\underline{v}^j) + \int_{\underline{v}^j}^{v^j} \bar{v}_o^j(s)ds,\ \forall v^j$.*

Theorem 4 gives us a way to construct a mechanism that is BIC in linear environment and we will design the IPPP project using this theorem in the following section.

CHAPTER 4

# MECHANISMS IN IPPP CONTEXT

## 4.1 Designing an indirect mechanism

As we pointed out in section 1 and section 2, IPPP project could be model in a Stackelberg game with two levels, where private parties are leaders and public sectors are followers. Government acts as mechanism designer in the process. Before the game, government first announce the payment (could be positive or negative) for each players according to different outcome $p^i_l(\mathbf{v},\mathbf{f}) > 0$ and $p^j_o(\mathbf{v},\mathbf{f}) < 0$, where $\mathbf{v}$ is the toll rates on highways and $\mathbf{f}$ is the flow in the network. Then, private parties and public sectors are revealed with their 'types': maintenance cost $c^i \in C$ and value of time (VOT) $v^j \in V$. The private parties first decide the toll rate $t$ on the highways they are managing to maximize their profits:

$$t^{i,*}_c = \arg\max_{t\in T^i} \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}} \left\{ I^i_l(t,\mathbf{t}^{-i},\mathbf{f}^*_{\mathbf{v},\mathbf{t}}) - \Omega(c^i,\mathbf{f}^*_{\mathbf{v},\mathbf{t}}) - p^i_l(t,\mathbf{t}^{-i},\mathbf{f}^*_{v,t}) - U^i_l \right\}, \qquad i = 1,\ldots,M$$

where $I^i_l(t,f)$ is the income from toll, $\Omega(c,f)$ is the maintenance cost, $f^*$ is the optimal responses of public sectors to the toll rates, and $U^i_l$ is the one-time investment and its possible return if private sector invest it elsewhere. Observing the toll rate on each road, public sectors make route choice decisions by minimizing their own travel costs:

$$f^{j,*}_{v,\mathbf{t}} = \arg\max_{f\in F^j} \mathbf{E}_{\mathbf{v}^{-j}} \left\{ -p^j_o(\mathbf{t},\mathbf{f}^{-j},f^j) - I^j_o(\mathbf{t},\mathbf{f}^{-j},f^j) - \Theta(v^j,f^j,\mathbf{f}^{-j}) + U^j_o \right\}, \qquad j = 1,\ldots,N$$

where $I_o^j$ is the payment for toll from public sector to private sector, $\Theta(f, v)$ is total monetized travel cost, and $U_o^j$ is the total travel cost without IPPP. After these are finished, the toll rate $\mathbf{t}$ and travel amount $\mathbf{f}$ are observed and government charge private party $i$ with $p_l^i(\mathbf{t}, \mathbf{f})$ and pay public sector $j$ with $p_o^i(\mathbf{t}, \mathbf{f})$.

As in problemeqs. (2.4a–2.4c), we want to implement the mechanism with Bayesian incentive compatibility and interim individual rationality. Then all the players' utility functions are maximized when all of them make social optimal decision. Also we want to ensure that, even under worst condition, the solution of problemeqs. (2.4a–2.4c) is larger than 0 so that no agent will deviate from the project. So the problem becomes:

$$\max_{\substack{\mathbf{p}(\mathbf{t},\mathbf{f}):\\ \mathbf{t}\in\mathbb{T}\\ \mathbf{f}\in\mathbb{F}}} \quad \mathbf{E}_{c,v}\left[\sum_i u_l^i(\mathbf{t_c}, \mathbf{f}^*_{\mathbf{v},\mathbf{t}}|\mathbf{c}) + \sum_j u_o^j(\mathbf{t}, \mathbf{f}_{\mathbf{v},\mathbf{t}}|\mathbf{v})\right] \tag{4.1a}$$

$$\begin{aligned} s.t. \quad & \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}}\left\{I_l^i(\tau_c^i, \boldsymbol{\tau}^{-i}_{\mathbf{c}^{-i}}, \mathbf{f}^*_{\mathbf{v},\tau_c}) - \Omega(c, \mathbf{f}^*_{\mathbf{v},\tau_c}) - p_l^i(\tau_c^i, \boldsymbol{\tau}^{-i}_{\mathbf{c}^{-i}}, \mathbf{f}^*_{\mathbf{v},\tau_\mathbf{c}})\right\} \\ & \qquad \geqslant \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}}\left\{I_l^i(t, \boldsymbol{\tau}^{-i}_{\mathbf{c}^{-i}}, \mathbf{f}^*_{\mathbf{v},\tau_c}) - \Omega(c, \mathbf{f}^*_{\mathbf{v},\tau_c}) - p_l^i(t, \boldsymbol{\tau}^{-i}_{\mathbf{c}^{-i}}, \mathbf{f}^*_{\mathbf{v},\tau_\mathbf{c}})\right\} \\ & \qquad \forall\ i = 1, \ldots, M,\ t \in T^i,\ \ c \in C^i, \end{aligned} \tag{4.1b}$$

$$\begin{aligned} & \mathbf{E}_{\mathbf{v}^{-j}}\left\{-p_o^j(\boldsymbol{\tau}_\mathbf{c}, \boldsymbol{\phi}^{-j}_{\mathbf{v}^{-j}}, \phi_v^j) - I_o^j(\boldsymbol{\tau}_\mathbf{c}, \boldsymbol{\phi}^{-j}_{\mathbf{v}^{-j}}, \phi_v^j) - \Theta(v, \phi_v^j, \boldsymbol{\phi}^{-j}_{\mathbf{v}^{-j}})\right\} \\ & \qquad \geqslant \mathbf{E}_{\mathbf{v}^{-j}}\left\{-p_o^j(\boldsymbol{\tau}_\mathbf{c}, \boldsymbol{\phi}^{-j}_{\mathbf{v}^{-j}}, f) - I_o^j(\boldsymbol{\tau}_\mathbf{c}, \boldsymbol{\phi}^{-j}_{\mathbf{v}^{-j}}, f) - \Theta(v, f, \boldsymbol{\phi}^{-j}_{\mathbf{v}^{-j}})\right\} \\ & \qquad \forall\ j = 1, \ldots, N,\ \mathbf{c} \in \mathbb{C},\ v \in V^j,\ f \in F^j, \end{aligned} \tag{4.1c}$$

$$\begin{aligned} & \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}}\left\{I_l^i(\tau_c^i, \boldsymbol{\tau}^{-i}_{\mathbf{c}^{-i}}, \mathbf{f}^*_{\mathbf{v},\tau_c}) - M(c, \mathbf{f}^*_{\mathbf{v},\tau_c}) - p_l^i(\tau_c^i, \boldsymbol{\tau}^{-i}_{\mathbf{c}^{-i}}, \mathbf{f}^*_{\mathbf{v},\tau_\mathbf{c}}) - U_l^i\right\} \geqslant 0 \\ & \qquad \forall\ i = 1, \ldots, M,\ c \in C, \end{aligned} \tag{4.1d}$$

$$\begin{aligned} & \mathbf{E}_{\mathbf{v}^{-j}}\left\{-p_o^j(\boldsymbol{\tau}_\mathbf{c}, \boldsymbol{\phi}^{-j}_{\mathbf{v}^{-j}}, \phi_v^j) - I_o^j(\boldsymbol{\tau}_\mathbf{c}, \boldsymbol{\phi}^{-j}_{\mathbf{v}^{-j}}, \phi_v^j) - \Theta(v, \phi_v^j, \boldsymbol{\phi}^{-j}_{\mathbf{v}^{-j}}) + U_o^j\right\} \geqslant 0 \\ & \qquad \forall\ j = 1, \ldots, N,\ \mathbf{c} \in \mathbb{C},\ v \in V^j, \end{aligned} \tag{4.1e}$$

The objective functioneq. (4.1a) is the total utility of all agents, i.e. the social welfare. The inequalityeq. (4.1b) andeq. (4.1c) denote the Bayesian incentive compatibility. The inequalitieseq. (4.1d) andeq. (4.1e) ensure that outcomes from IPPP project is better than deviation, where the deviated agent gets 0. Let $g_i$ denote the indirect mechanism we formulated above.

## 4.2 Equivalent direct mechanism

Through the proof of Revelation Principle, we could prove that this mechanism is equivalent to a direct mechanism in MLMF Stackelberg game. Thus we have the following definition.

**Definition 3.** ***Equivalent direct mechanism to problemeqs. (4.1a–4.1e)***. *For an indirect mechanism described in problemeqs. (4.1a–4.1c), there is an equivalent direct mechanism $g_d$ in which there are $M$ leaders and $N$ followers with type spaces $C$ and $V$ respectively. Players report their types $\tilde{c}$ and $\tilde{v}$ in a Stackelberg game. Then the policy maker decides the allocation and payment. For leader $i$, the allocation is:*

$$\sigma_l^i(\tilde{c}) = \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}}\left[-\Omega(\tilde{c}, \mathbf{f}^*_{\mathbf{v},\tau_{\tilde{c}}})\right], \tag{4.2}$$

*and the payment to government is*

$$\mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}}\left[I_l^i(\tau^i_{\tilde{c}}, \boldsymbol{\tau}^{-i}_{\mathbf{c}^{-i}}, \mathbf{f}^*_{\mathbf{v},\tau_{\tilde{c}}}) + p_l^i(\tau^i_{\tilde{c}}, \boldsymbol{\tau}^{-i}_{\tilde{\mathbf{c}}^{-i}}, \mathbf{f}^*_{\mathbf{v},\tau_{\tilde{\mathbf{c}}}}) + U_l^i\right]$$

*For the follower $j$, the allocation is*

$$\sigma_o^j(\tilde{v}) = \mathbf{E}_{\mathbf{v}^{-j}}\left[-\Theta(\tilde{v}, \phi^j_{\tilde{v}}, \boldsymbol{\phi}^{-j}_{\mathbf{v}^{-j}})\right], \tag{4.3}$$

*and the payment to government is*

$$\mathbf{E}_{\mathbf{v}^{-j}}\left[-I_o^j(\boldsymbol{\tau}_{\mathbf{c}}, \boldsymbol{\phi}^{-j}_{\mathbf{v}^{-j}}, \phi^j_{\tilde{v}}) + p_o^j(\boldsymbol{\tau}_{\mathbf{c}}, \boldsymbol{\phi}^{-j}_{\mathbf{v}^{-j}}, \phi^j_{v}) - U_o^j\right]$$

In the equivalent direct mechanism, all the players report their types and the amount of 'Goods' allocated is determined by the strategy function of each player ineqs. (4.1a–4.1e).

In mechanism $g_d$, the utility function of each player is not changed, but we redefine the allocation and money transfer functions. We could have the following lemma:

**Lemma 5.** *(Linear environment) In mechanism $g_d$, if the utility functions satisfy the following assumptions, then it is a linear environment.*

- $\Omega(c, f) = c \cdot \omega(c, f)$,
- $\Theta(v, f) = v \cdot \theta(v, f)$,
- *$C_i$ and $V_j$ are bounded intervals, $\forall\ i, j$,*
- *All $c_i$ and $v_j$ are independent,*
- *The distribution function of $c_i$ on $C_i$ and $v_j$ on $V_j$ is strictly positive,*

Proof is straightforward through Definition 2. Therefore, according to Theorem 4, in a linear environment the utility functions of leaders and followers should satisfy,

$$u_l^i(g_d|c) = u_l^i(g_d|\underline{c}) - \int_{\underline{c}}^{c} \omega(s, \mathbf{f})ds, \tag{4.4a}$$

$$u_o^j(g_d|v) = u_o^j(g_d|\underline{v}) - \int_{\underline{v}}^{v} \theta(s, \mathbf{f})ds. \tag{4.4b}$$

Then we could get the possible money transfer that satisfies Bayesian incentive compatibility. Notice that the utility functions of leaders and followers are calculated by

$$u_l^i(\mathbf{t},\mathbf{f}|c) = \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}}\left[-c\,\omega(c,\mathbf{f}) + I_l^i(\mathbf{t},\mathbf{f}) - p_l^i(\mathbf{t},\mathbf{f}) - U_l^i\right] \tag{4.5a}$$

$$u_o^j(\mathbf{t},\mathbf{f}|v) = \mathbf{E}_{\mathbf{v}^{-j}}\left[-v\,\theta(v,\mathbf{f}) - I_o^j(\mathbf{t},\mathbf{f}) - p_o^j(\mathbf{t},\mathbf{f}) + U_o^j\right] \tag{4.5b}$$

From (4.4a) and (4.4b), we could get the payment from private sector $i$ to the government:

$$p_l^i(\mathbf{t},\mathbf{f}) = -u_l^i(\mathbf{t},\mathbf{f}|\underline{c}) + \int_{\underline{c}}^{c} \omega(s,f)ds - \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}}\left[c\,\omega(c,\mathbf{f}) - I_l^i(\mathbf{t},\mathbf{f})\right] - U_l^i$$

Similarly, from (4.4b) and (4.4b), we could get the payment from the public sector $j$ to the government:

$$p_o^j(\mathbf{t},\mathbf{f}) = -u_o^j(\mathbf{t},\mathbf{f}|\underline{v}) + \int_{\underline{v}}^{v} \theta(s,f)ds - \mathbf{E}_{\mathbf{v}^{-j}}\left[v\,\theta(v,\mathbf{f}) + I_o^j(\mathbf{t},\mathbf{f})\right] + U_o^j$$

## 4.3 Social optimum and budget balance

If the government set the social optimal toll rate for private party and social optimal flow for public party (although it is not practical), then the total payment under social optimal should be (for simplicity we write $\boldsymbol{\tau}_{\mathbf{c},\mathbf{v}}$ as $\boldsymbol{\tau}$ and $\boldsymbol{\phi}_{\mathbf{c},\mathbf{v}}$ as $\boldsymbol{\phi}$)

$$
\begin{aligned}
\sum_i p_l^i(\boldsymbol{\tau},\boldsymbol{\phi}) + \sum_j p_o^j(\boldsymbol{\tau},\boldsymbol{\phi}) = & \sum_i \left\{-u_l^i(\boldsymbol{\tau},\boldsymbol{\phi}|\underline{c}) + \int_{\underline{c}}^{c} \omega(s,\boldsymbol{\phi})ds - \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}}\left[c\,\omega(c,\boldsymbol{\phi}) - I_l^i(\boldsymbol{\tau},\boldsymbol{\phi})\right] - U_l^i\right\} \\
& + \sum_j \left\{u_o^j(\boldsymbol{\tau},\boldsymbol{\phi}|\underline{v}) + \int_{\underline{v}}^{v} \theta(s,\boldsymbol{\phi})ds - \mathbf{E}_{\mathbf{v}^{-j}}\left[v\,\theta(v,\boldsymbol{\phi}) + I_o^j(\boldsymbol{\tau},\boldsymbol{\phi})\right] + U_o^j\right\} \\
= & -\sum_i \left\{u_l^i(\boldsymbol{\tau},\boldsymbol{\phi}|\underline{c}) - \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}}\left[\int_{\underline{c}}^{c} \omega(s,\boldsymbol{\phi})ds\right]\right\} \\
& - \sum_j \left\{u_o^j(\boldsymbol{\tau},\boldsymbol{\phi}|\underline{v}) - \mathbf{E}_{\mathbf{v}^{-j}}\left[\int_{\underline{v}}^{v} \theta(s,\boldsymbol{\phi})ds\right]\right\} \\
& - \sum_i \left\{\mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}}\left[c\omega(c,\boldsymbol{\phi})\right] + U_l^i\right\} + \sum_j \left\{-\mathbf{E}_{\mathbf{v}^{-j}}\left[v\theta(v,\boldsymbol{\phi})\right] + U_o^j\right\} \\
& + \sum_i \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}}\left[I_l^i(\boldsymbol{\tau},\boldsymbol{\phi})\right] - \sum_j \mathbf{E}_{\mathbf{v}^{-j}}\left[I_o^j(\boldsymbol{\tau},\boldsymbol{\phi})\right] \\
= & -\text{Total utility of private sectors} - \text{Total utility of public sectors} \\
& - \text{total cost for private sectors} \\
& + \text{Total travel cost improvement for public sectors} + \mathcal{I}
\end{aligned}
\tag{4.8}
$$

Where $\mathcal{I} = \sum_i \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}}\left[I_l^i(\boldsymbol{\tau},\boldsymbol{\phi})\right] - \sum_j \mathbf{E}_{\mathbf{v}^{-j}}\left[I_o^j(\boldsymbol{\tau},\boldsymbol{\phi})\right]$. In the following sections, we will discuss on different $\mathcal{I}$ to see what (4.8) is telling us.

### 4.3.1 $\mathcal{I} \approx 0$

This condition will be true when number of private sectors, $N$, is really large. As the type of each private sector is independently distributed, the interim expectation of total toll income will be close to *ex post* total toll income. Then it is easy to get the total expense of government is

$$\begin{aligned}\text{Government's total expense} &= \text{Total utility of private sectors} + \text{Total utility of public sectors} \\ &\quad + \text{Total cost for private sectors} \\ &\quad - \text{Total travel cost improvement for public sectors}\end{aligned} \tag{4.9}$$

It is obviously that the the first three elements of (4.9) is positive and total travel cost improvement should less than total utility of public sectors. (Otherwise the cost of toll will be larger than total travel cost without IPPP which is unrealistic.) This indicates that government should pay for social optimum, while the total amount is less than the government needs to pay without an IPPP.

Then we could get the total utility of all sectors when it is budget-balanced, that is Government's total expense equals 0:

$$\begin{aligned}\text{Total utility when budget is balanced} &= \text{Total travel cost improvement for public sectors} \\ &\quad - \text{Total cost for private sectors}\end{aligned}$$

So that as long as the improvement of total travel cost is larger than total maintenance and construction cost, the IPPP project could be both budget-balanced and interim Pareto efficient.

### 4.3.2 $\mathcal{I} \neq 0$

When $\mathcal{I} \neq 0$, the interim expectation of toll income is different from the *ex post* income. This could due to the number of private sector, $N$, is small or an increase or decrease of maintenance cost among all private sectors. Notice that when the realization of the cost $\mathbf{c}$ is relatively low, we will have

$$\sum_i \mathbf{E}_{\mathbf{v},\mathbf{c}^{-i}} \left[ I_l^i(\boldsymbol{\tau}, \boldsymbol{\phi}) \right] > \sum_j \mathbf{E}_{\mathbf{v}^{-j}} \left[ I_o^j(\boldsymbol{\tau}, \boldsymbol{\phi}) \right],$$

and vice versa. This means that when the maintenance cost is low, the government could pay less for achieving social optimum, whereas when the cost is high, the government should compensate the private sectors more. Thus improvement in road construction and maintenance technology and reduction of maintenance cost is a win-win situation for both government and private sectors. This gives government incentives to encourage and support private company to invest more in research and development (R&D).

## 4.4 Can government achieve social optimum for free?

From the discussion in section 4.3.1 and 4.3.2, we could have the following equation

$$\begin{aligned}\text{Government's total expense} = &\text{Total net social benefit} + \sum_{\substack{\text{Private}\\\text{sectors}}} \text{one-time and maintenance cost}\\ &- \sum_{\substack{\text{Public}\\\text{sectors}}} \text{Travel cost improvement} - \mathcal{I}\end{aligned}$$

If it is budget balanced, the government's expense equals 0 and then the total net social benefit should become

$$\text{Total social net benefit} = \sum_{\substack{\text{Public}\\\text{sectors}}} \text{Travel cost improvement} - \sum_{\substack{\text{Private}\\\text{sectors}}} \text{one-time and maintenance cost} + \tag{4.10}$$

At the worst case, for example the maintenance cost for all private sectors are extremely high. Let $-\mathcal{I}_w$ indicates the value of $\mathcal{I}$ at this time. Then the total social net benefit becomes

$$\sum_{\substack{\text{Public}\\\text{sectors}}} \text{Travel cost improvement} - \sum_{\substack{\text{Private}\\\text{sectors}}} \text{one-time and maintenance cost} - \mathcal{I}_w \tag{4.11}$$

Thus we have the following theorem:

**Theorem 6.** *The government could achieve social optimum and budget balance at the same time if and only if (4.11)$\geq 0$.*

***Remark 1.*** (4.11) could be generalized as

$$\begin{aligned}\text{Total social net beneft} &= \text{Total cost improvement or benefit} - \text{Total one-time and recurrent cost}\\ &\quad - \text{Difference between interim and ex post expected income at worst case}\end{aligned} \tag{4.12}$$

This shows that in analyzing the viability of IPPP (or PPP) projects, it is not sufficient to conduct benefit-cost analysis, because the net benefit of the society does not simply equal to "benefit minus cost", but is also related to the difference in interim expected toll income and ex post toll income. This consideration is extremely important to avoid renegotiation brought by cost overruns.

***Remark 2.*** Theorem 6 gives us a way to evaluate and determine whether a potential project could achieve both social optimal and budget balance. This is really necessary given some, if not a lot, examples that projects need to be renegotiated.

***Remark 3.*** If (4.11) $<$ 0, the government should trade off between budget balance and social optimum. For example, the government could sacrifice the utility of private sectors or public sectors to keep budget balanced. Or, which we would recommend, the government could pay a limit amount money to secure the social optimum. Notice that without IPPP, the government should pay

$$\sum_{\substack{\text{Private}\\\text{sectors}}} \text{one-time and maintenance cost}$$

to get the same level of service. Thus, as long as government's total expense is less than this amount, the IPPP could still be a Pareto improvement. The total social net benefit becomes

$$
\begin{aligned}
0 &< \text{Total social net benefit} \\
&= \sum_{\substack{\text{Public} \\ \text{sectors}}} \text{Travel cost improvement} - \sum_{\substack{\text{Private} \\ \text{sectors}}} \text{one-time and maintenance cost} \\
&\qquad - \mathcal{I}_w + \text{Government's total expense} \\
&< \sum_{\substack{\text{Public} \\ \text{sectors}}} \text{Travel cost improvement} - \mathcal{I}_w.
\end{aligned}
$$

***Remark 4.*** If (4.11) $< \sum_{\substack{\text{Private} \\ \text{sectors}}}$ one-time and maintenance cost, then the government needs to pay more than it needs to pay under public provision, that means IPPP is not suitable for government to limit its expense budget.

We will illustrate the above results through a numerical example in section 5. Then we will compare different forms of PPP projects together.

CHAPTER 5

# PROJECT IN SIOUX-FALLS NETWORK

Sioux-Falls Network is a 24-zone, 24-node and 76-link road network. It is widely used in traffic assignment problem [17]. All the related files used in this paper could be accessed through [2]. The map of Sioux-Falls network is shown in Figure 5.1. In this section, we first verify the feasibility of IPPP project on Sioux-Falls network, then find out the optimal implementation, and at last illustrate the performance of the project.

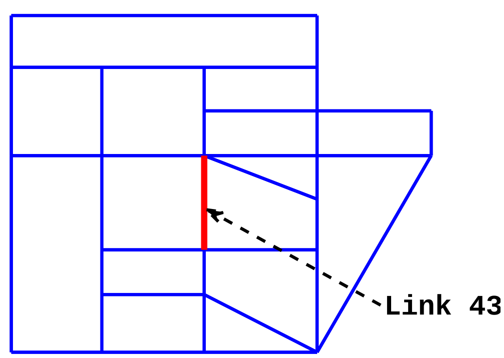


Figure 5.1: Sioux-Falls Network

For simplicity, we assume that there is only one private party and one public party in the problem. There is only one link that is leased to the private party (link 58, from node 15 to node 10, marked in Figure 5.1). This is a special case of the model we discuss here, but the result could be easily generalized to multi-player case.

Table 5.1: Feasibility under different scenarios

| Scenarios | Low Cost | Medium Cost | High Cost |
|---|---|---|---|
| Total Travel Time Improvement ($M$\$) | 3.4124 | 3.1300 | 1.9640 |
| Expected Cost ($M$\$) | 0.6155 | 1.2305 | 1.8457 |
| Risk Factor ($M$\$) | 1.6561 | 1.6311 | 1.2269 |
| Improvement - Cost - Risk ($M$\$) | 1.1408 | 0.2684 | $-1.1086$ |

## 5.1 Social optimal and budget-balanced

A PPP project is social optimal and budget-balanced, as it is pointed out in Theorem 6 in Section 4.4, when total travel cost improvement is larger than the sum of total one-time and maintenance cost and worst risk factor. If this condition is satisfied, the private and public party will not deviate from the game. The value of these three factors under different scenarios is shown in Table 5.1. It is obvious that in low cost and medium cost scenarios, the condition of Theorem 6 is satisfied. However in high cost scenario, the inequality is not satisfied. From the discussion of Theorem 6, although we could not achieve social optimal and budget-balanced project at the same time, IPPP is still beneficial because government could pay 1.1086 million dollars to achieve social optimal, which is less than 1.8457 million dollars. Thus we could conclude that IPPP should be implemented in the network.

## 5.2 Evaluation

To evaluate the merit of utilizing IPPP in infrastructure management, we compare the results of implementing IPPP with those of other three approaches. We implement IPPP, original IPPP and PPP on the same link in the same network. The costs and VOTs are the same and private sectors in three scenarios could choose toll rates from the same set. Net benefits are calculated by the utility of the sector in current scenario subtracted by the utility of the same sector under government provision. The simulation result is shown in Figure 5.2.

It is obviously that all three approaches could achieve *ex ante* potential Pareto-improvement compared with government provision. Notice that they all have risks to make private sector to get negative net benefit, which means that they all have chances to end up with project failures or deviation of private sectors. But it is noteworthy that improved IPPP and PPP has relatively less chance to fail (the amounts of failure times are smaller, see Table 5.2) and less loss when come to failures (as marked by dashed line on Figure 5.2). Another apparent trend is that both improved IPPP and original IPPP guarantee higher net benefit of public sector than PPP. This shows the advantages of improved IPPP we mentioned in section 1 - It provides better public support, better regulatory on road pricing than PPP and lower financial risk and better regulatory than original IPPP.

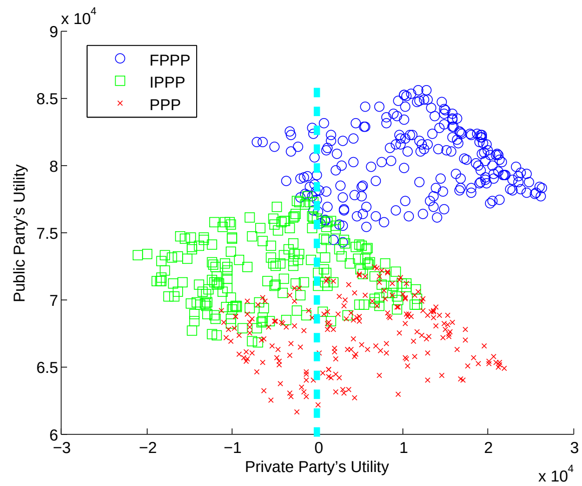


Figure 5.2: The net benefit of public and private sector in implementation of improved IPPP, original IPPP and PPP

Table 5.2: Failure time of three approaches

| Total number of simulation | Improved IPPP | Original IPPP | PPP |
|---|---|---|---|
| 200 each | 24 Failures | 139 Failures | 63 Failures |

# CHAPTER 6
## CONCLUSION

In urban transportation, the brownfield highway project, which aims at managing existing highways, is difficult to be implemented and to get best result. The current government provision way has many disadvantages, such as insufficient fund, relatively high operating and maintaining cost, and tax distortion. The prevailing Public-Private Partnership (PPP), is widely used to overcome these shortcomings. However, whether PPP could provide government social optimum and budget balance at the same time is still questioned. Moreover, regulating the private party on road pricing is another problem [23]. Thus [10] proposed that return part of the revenue from the road pricing to gain public support. However, [10] do not provide direct proof or example to support this proposal.

In this paper, we presented a method to model, implement and evaluate the improved Investment Public-Private Partnership (IPPP) through mechanism design in a multi-leader-multi-follow (MLMF) Stackelberg game. We derived and proved the theories of implementability in dominant strategy (IDS) equilibrium in Stackelberg game when the type space is continuous or discrete. Then we used these theories to develop our modelEquations (4.1a–4.1e), and proved its feasibility. We applied the model to Sioux-Falls network for illustration.

The improved IPPP in our paper is different from the one in [10] in the following way: in the improved IPPP, the rent rates that government charges from the private sectors are floating according to the decision made by private and public sectors. This could help government to regulate the road pricing and at the same time reduce the risk when the maintenance cost is extremely high and

Table 6.1: Advantages of improved IPPP over following management forms

| | Potential Pareto-improvement | Relief of tax distortion | Public support | Better regulatory on road pricing | Lower financial risk |
|---|---|---|---|---|---|
| Government provision | √ | √ | - | √ | - |
| PPP | - | - | √ | √ | - |
| Original IP PP | - | - | - | √ | √ |

harmful to the financial viability of the project. From our analysis and results, we could claim that our improved IPPP has following advantages compared with government provision, PPP and original IPPP, shown in Table 6.1:

Besides answering the question, whether social optimal PPP is free or not, our paper contributes in both methodology side and application side. On methodology side, we derived the implementability of dominant strategy equilibrium and Bayesian equilibrium in multi-leader-multi-follower Stackelberg game and apply the results in proof of feasibility of IPPP. On application side, our paper is the first to apply mechanism design in transportation infrastructure management context. Similar method and idea could be used in management of other infrastructures, such as tunnel, bridge and power grid where private information is held by agents.

Several future work directions are interesting and promising. Our model is a one stage model, thus the interaction between different stages and information update is not taken into consideration. With tools in dynamic mechanism design, the model could be more precise if it is developed as a multi-stage one.

Another possible direction we would suggest is that when one considers multi-modal transportation, such as including bus or public transit into the model, IPPP has an additional advantage that it could help the development of public transportation and reduce of emission. The effect of IPPP on such side would be significant.

Others, such as incorporate more sophisticated and accurate road pricing and maintenance model, could also be worth of investigation.

# APPENDIX A

## NOMENCLATURE

## NOMENCLATURE

$\mathbf{\Phi}^*$ The set of all social optimal flow $\boldsymbol{\phi}_{\mathbf{v},\mathbf{t}}$

$\boldsymbol{\phi}_{\mathbf{v},\mathbf{t}}$ The optimal flow when value of time is $\mathbf{v}$ and toll rate is $\mathbf{t}$

$\boldsymbol{\tau}_{\mathbf{c}}$ The social optimal toll rate when maintenance cost is $\mathbf{c}$

$\Gamma$ The set of all outcomes in a Stackelberg game

$\mathbb{C}$ The type space of all leaders/private sectors; $\mathbb{C} = C^1 \times \cdots \times C^N$

$\mathbb{V}$ The type space of all followers/ public sectors; $\mathbb{V} = V^1 \times \cdots \times V^M$

$\mathbf{f}^{-j}$ The flow of other public sectors except $j$

$\mathbf{Pr}(\cdot)$ The probability density distribution of types of all agents

$\mathbf{T}^*$ The set of all social optimal toll rate $\boldsymbol{\tau}_{\mathbf{c}}$

$\mathbf{t}^{-i}$ The toll rate of other private sectors except $i$

$v_l^i(\cdot)$ Part of utility function of leader in linear environment

$\Omega(c, \mathbf{f})$ Maintenance cost expenditure for private party

$\Theta(v, \mathbf{f})$ Total time loss for public sector

$\tilde{\mathbf{c}}$ Reported types of leaders/private sectors

$\tilde{\mathbf{v}}$ Reported types of followers/ public sectors

$C$ The set of all possible maintenance cost $c$

$c^i$ Maintenance cost and type of leader $i$

$F$ The set of all possible flow $f$

$f^j$ Flow on the road travelled by follower $j$ and strategy of follower $j$

$f_{v,\mathbf{t}}^{j,*}$ The best strategy of public sector $j$ given the value of time $v$ and toll rate $\mathbf{t}$

$g$ The outcome function

$g_d$ The direct mechanism equivalent to $g_i$

$g_i$ The indirect mechanism in IPPP context

$I(v, \mathbf{f})$ toll revenue for private party

$M$ Number of private sectors

$m_l^i$ Initial endowment for the leader

$m_o^j$ Initial endowment for the follower

$N$ Number of public sectors

$p_l^i$ Payment function of leader/private sector $i$

$p_o^j$ Payment function of follower/public sector $j$

$SW(\cdot)$ Social welfare function

$T$ Set of all possible toll $t$

$t^i$ Toll rate on the road leased to private party $i$ and strategy of leader $i$

$t_c^{i,*}$ The optimal toll rate for private party $i$ with maintenance cost $c$

$U_l^i$ Private sector $i$'s one-time investment to IPPP projects and its possible return if invested elsewhere

$u_l^i$ Utility function of leader (private sector) $i$

$U_o^j$ Total travel cost for public sector $j$ without IPPP

$u_o^j$ Utility function of follower (public sector) $j$

$V$ The set of all possible value of time $v$

$v^j$ Value of time and type of follower $j$

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